Parameterized Complexity of MinCSP over the Point Algebra

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Constraint Satisfaction Problem (CSP) over Point Algebra

Point Algebra is the set of relations $\{<, >, \leq, \geq, =, \neq\}$ over $\mathbb{Q}$.
Let us fix a *constraint language* $\Gamma \subseteq \{<, >, \leq, \geq, =, \neq\}$.

$\mathrm{CSP}(\Gamma)$ input is $(V, \mathcal{C})$, where

- V is a set of variables,
- $\mathcal{C}$ is a set of constraints of the form $u \odot v$ with $u, v \in V$ and $\odot \in \Gamma$.

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Ex:

$$v_1 \leq v_2, \qquad v_1 \neq v_2, \qquad v_2 > v_3$$

is satisfiable, e.g. $\alpha(v_1) = 1$, $\alpha(v_2) = 2$, $\alpha(v_3) = 0$.

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$$v_1 < v_2, \qquad v_2 = v_3, \qquad v_1 > v_3$$

is not satisfiable.

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$\mathrm{CSP}(\Gamma)$ is solvable in polynomial time for all $\Gamma \subseteq \mathcal{A}$.

Minimum-Cost CSP (MinCSP) over Point Algebra

$\textsc{MinCSP}(\Gamma)$ input is $(V, \mathcal{C}, k)$, where

- V is a set of variables,
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- k is an integer.

Q: is there $\alpha : V \to \mathbb{Q}$ that *breaks* at most k constraints in $\mathcal{C}$?

Unless $\textsc{MinCSP}(\Gamma)$ is trivial, it is NP-hard and UGC-hard to $O(1)$-apx.

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We study parameterized complexity:

FPT	vs	W[1]-hard
$f(k) \cdot n^{O(1)}$	vs	$n^{g(k)}$

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Theorem

Let $\Gamma \subseteq \{<, >, \leq, \geq, =, \neq\}$.

- *If* $\{\leq, \neq\} \subseteq \Gamma$, *then* $\textsc{MinCSP}(\Gamma)$ *is W[1]-hard.*
- *If* $\{\geq, \neq\} \subseteq \Gamma$, *then* $\textsc{MinCSP}(\Gamma)$ *is W[1]-hard.*
- *Otherwise,* $\textsc{MinCSP}(\Gamma)$ *is FPT.*

Motivation

For different $\Gamma \subseteq \{<, >, \leq, \geq, =, \neq\}$, $\textsc{MinCSP}(\Gamma)$ captures Multicut, Directed (Subset) Feedback Arc Set and Directed Symmetric Multicut.

- $\Gamma = \{<\}$: *Directed Feedback Arc Set* is FPT (Chen Liu Lu O'Sullivan Razgon @STOC'08 $\rightarrow$ JACM'08)

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- New flow-augmentation based algorithms for the first three, last still open (Kim Masařík Pilipczuk Sharma Wahlström SIDMA'24)

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We show that Directed Symmetric Multicut is W[1]-hard, and every $\textsc{MinCSP}(\Gamma)$ that avoids it is FPT.

MinCSP and Graph Problems

Directed Feedback Arc Set and MinCSP($<$)

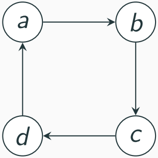
An instance of $\mathrm{CSP}(<)$ is satisfiable iff the digraph is acyclic.

$a < b$

$b < c$

$c < d$

$d < a$



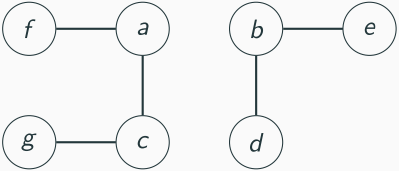
$\textsc{MinCSP}(<)$ is Directed Feedback Arc Set: given a digraph G and $k \in \mathbb{N}$, delete at most k arcs from G to make it acyclic.

Edge Multicut and MinCSP$(=, \neq)$

An instance of $\mathrm{CSP}(=, \neq)$ is satisfiable iff there is no $=$-path between disequal variables.

$a = c$
$a = f$
$b = d$
$b = e$
$c = g$

$a \neq b$
$c \neq d$
$e \neq f$
$f \neq g$



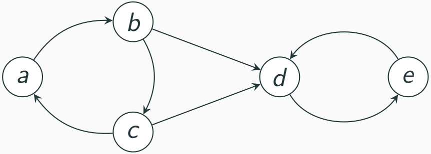
$\textsc{MinCSP}(=, \neq)$ is Edge Multicut[1]: given a graph G, a set $\mathcal{T} \subseteq \binom{V(G)}{2}$ and $k \in \mathbb{N}$, delete k edges from G so that no pair in $\mathcal{T}$ is connected.

[1] Simple trick allows assuming wlog that $\neq$-constraints are undeletable.

Directed Symmetric Multicut and MinCSP($\leq, \neq$)

An instance of $\mathrm{CSP}(\leq, \neq)$ is satisfiable iff there is no $\leq$-cycle with two disequal variables.

$$\begin{array}{ll} a \leq b & \\ b \leq c & \\ c \leq a & a \neq d \\ b \leq d & b \neq e \\ c \leq d & b \neq c \\ d \leq e & \\ e \leq d & \end{array}$$



$\mathrm{MinCSP}(\leq, \neq)$ is Directed Symmetric Multicut[2]: given a digraph G, a set $\mathcal{T} \subseteq \binom{V(G)}{2}$ and $k \in \mathbb{N}$, delete k arcs from G so that no pair in $\mathcal{T}$ is strongly connected.

[2]Simple trick allows assuming wlog that $\neq$-constraints are undeletable.

Classification

Classification: Case Analysis

$\Gamma \subseteq \{<, >, \leq, \geq, =, \neq\}$, so 2^6 cases.

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Lemma 1: If $\leq \in \Gamma$, then $\textsc{MinCSP}(\Gamma \cup \{=\}) \leq_{FPT} \textsc{MinCSP}(\Gamma)$.
Proof: Replace $u = v$ with $u \leq v$ and $v \leq u$. Opt deletes at most one.

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Lemma 2: If $\leq, \neq \in \Gamma$, then $\mathrm{MinCSP}(\Gamma \cup \{<\}) \leq_{FPT} \mathrm{MinCSP}(\Gamma)$.
Proof: Replace $u < v$ with $u \leq v$ and $u \neq v$. Opt deletes at most one.

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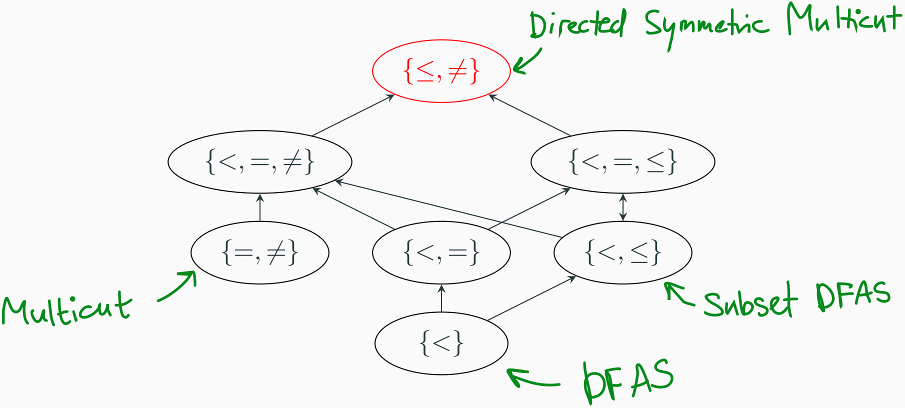
Lemmas 1 & 2 $\implies$ $\textsc{MinCSP}(<, \leq, =, \neq) \leq_{FPT} \textsc{MinCSP}(\leq, \neq)$.

Classification: Map



$\text{MinCSP}(\leq, \neq)$ is W[1]-hard.

$\text{MinCSP}(<, =, \leq) \leq_{FPT} \text{MinCSP}(<, \leq)$, which is FPT [CCHM].

$\text{MinCSP}(<, =, \neq)$ solved by reduction into Bundled Almost 2-Sat.

Algorithm

Almost 2-Sat

ALMOST 2-SAT input is $(V, \mathcal{C}, k)$, where

- V is a set of variables,
- $\mathcal{C}$ is a set of constraints of the form $(u \vee v)$, $(\bar{u} \vee \bar{v})$ or $(\bar{u} \vee v)$,
- k is an integer.

Q: is there $\alpha : V \to \{0, 1\}$ that *breaks* at most k constraints in $\mathcal{C}$?

ALMOST 2-SAT is FPT [Razgon O'Sullivan ICALP'08→JCSS'09]

Bundled Almost 2-Sat

In Bundled Almost 2-Sat, constraints are 2-Sat formulas, e.g.:

1. $(a \vee b) \wedge (\bar{c} \vee \bar{d})$
2. $(a \rightarrow b) \wedge (b \rightarrow c) \wedge (c \rightarrow d)$
3. $(a \vee b) \wedge (\bar{a} \vee \bar{b})$

Theorem (Kim Kratsch Pilipczuk Wahlström SODA'23)

Bundled Almost 2-Sat *is FPT*[3] *if all formulas are* $2K_2$*-free.*

[3] w.r.t. k + max formula length

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1. $(a \vee b) \wedge (\bar{c} \vee \bar{d})$
   Edges $\{ab, cd\}$: not $2K_2$-free
2. $(a \to b) \wedge (b \to c) \wedge (c \to d)$
   Edges $\{ab, bc, cd\}$: $2K_2$-free
3. $(a \vee b) \wedge (\bar{a} \vee \bar{b})$
   Edges $\{ab\}$: $2K_2$-free

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Bundled Almost 2-Sat *is FPT*[3] *if all formulas are $2K_2$-free.*

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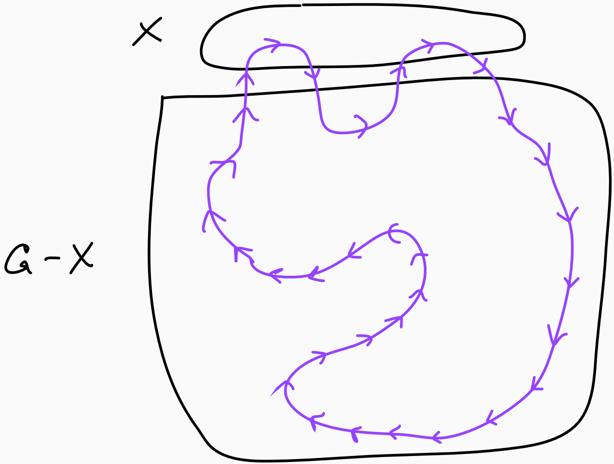
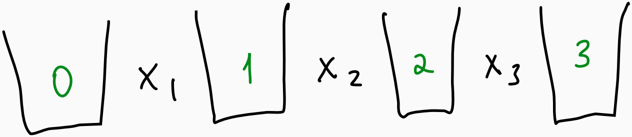
Algorithm for MinCSP($<$)

Iterative compression: $X \subseteq V$ s.t. $|X| \leq k + 1$ and $G - X$ is acyclic.



Algorithm for MinCSP($<$)

Let $\alpha : V \to \mathbb{Q}$ be opt. Guess $\alpha|_X$ in $2^{O(k \lg k)}$ time.



New goal: assign $V \setminus X$ into $|X| + 1$ buckets.

Introduce Boolean variables: $v \in V(G) \mapsto (v_0, \ldots, v_{|X|})$.

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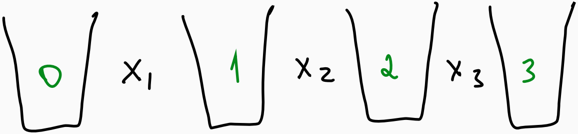

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Introduce Boolean variables: $v \in V(G) \mapsto (v_0, \ldots, v_{|X|})$.

Intepretation: $v_i = 1$ iff bucket$(v) \geq i$.

Ex: bucket$(v) = 3$, $v \mapsto (1, 1, 1, 1, 0, \ldots, 0)$.

Algorithm for MinCSP($<$)

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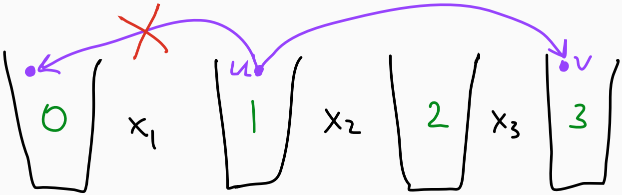


New goal: assign $V \setminus X$ into $|X| + 1$ buckets.

Introduce Boolean variables: $v \in V(G) \mapsto (v_0, \ldots, v_{|X|})$.

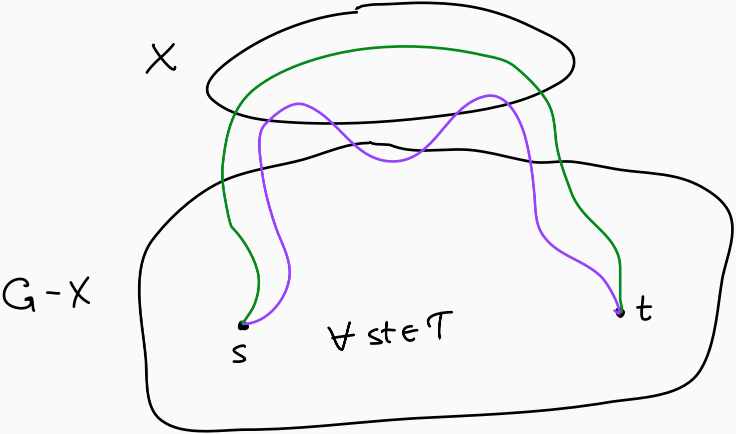
For every arc (u, v) in G, add constraint

$$\bigwedge_{i<j} (u_i \leftarrow u_j) \wedge \bigwedge_{i<j} (v_i \leftarrow v_j) \wedge \bigwedge_{i \leq j} (u_i \to v_j).$$

Algorithm for MinCSP$(=, \neq)$

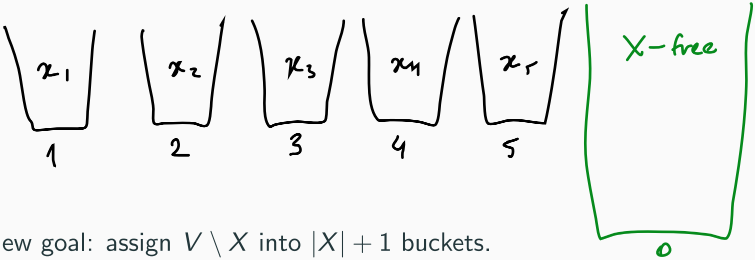
Input: graph G, set $\mathcal{T} \subseteq \binom{V}{2}$, integer k.

Compression: $X \subseteq V$ s.t. $|X| \in O(k)$ and X cuts all st-paths for $st \in \mathcal{T}$.



Algorithm for MinCSP$(=, \neq)$

Let $\alpha : V \to \mathbb{Q}$ be opt. Guess $\alpha|_X$ in $2^{O(k \log k)}$ time.



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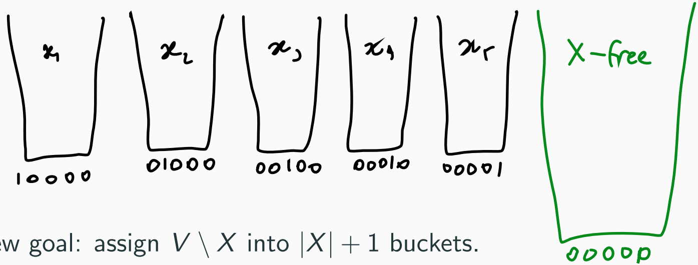

New goal: assign $V \setminus X$ into $|X| + 1$ buckets.

Introduce Boolean variables: $v \in V(G) \mapsto (v_1, \ldots, v_{|X|})$.

If bucket$(v) = 0$, then $v \mapsto (0, 0, 0, \ldots, 0)$.
If bucket$(v) = 2$, then $v \mapsto (0, 1, 0, \ldots, 0)$.

Algorithm for MinCSP$(=, \neq)$

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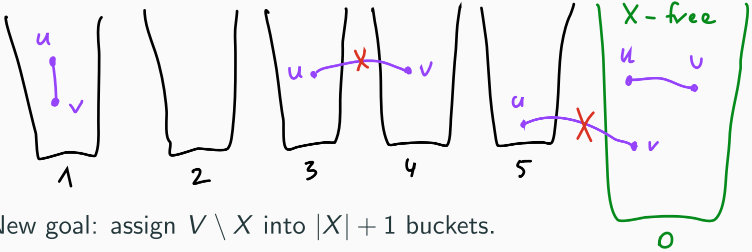


New goal: assign $V \setminus X$ into $|X| + 1$ buckets.

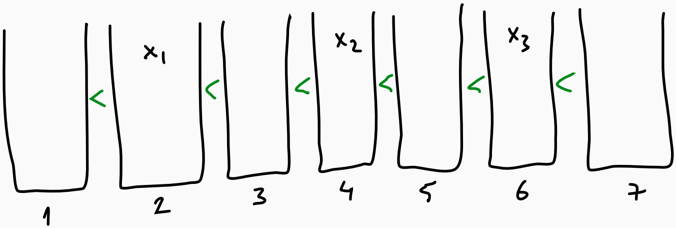
Introduce Boolean variables: $v \in V(G) \mapsto (v_1, \dots, v_{|X|})$.

For every edge uv in G, add constraint

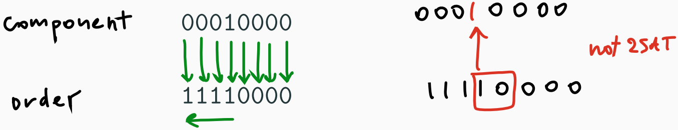
$$\underbrace{\bigwedge_{i<j} (\bar{u}_i \vee \bar{u}_j) \wedge \bigwedge_{i<j} (\bar{v}_i \vee \bar{v}_j)}_{\text{at most one } 1} \wedge \underbrace{\bigwedge_{i} (u_i \to v_i) \wedge \bigwedge_{i} (v_i \to u_i)}_{\vec{u} = \vec{v}}.$$

Algorithm for MinCSP$(<, =, \neq)$

Combine both encodings.



Crucial: $\text{component}(v) = i \implies \text{order}(v) = i$ can be enforced in $2K_2$-free 2-SAT, not other way around:

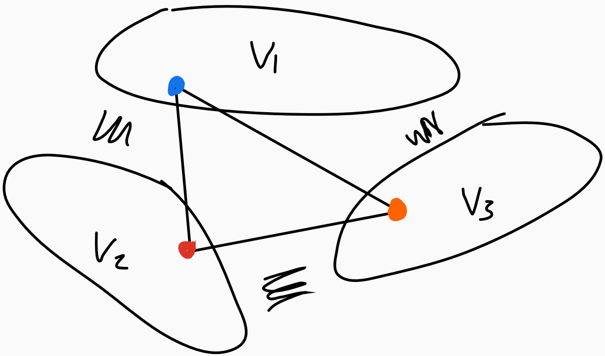


Lower Bound

W[1]-hardness of Directed Symmetric Multicut

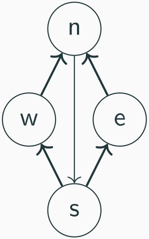
Directed Symmetric Multicut: given a digraph D, a set $\mathcal{T} \subseteq \binom{V}{2}$, and $k \in \mathbb{N}$, delete k arcs so that no pair in $\mathcal{T}$ is strongly connected.

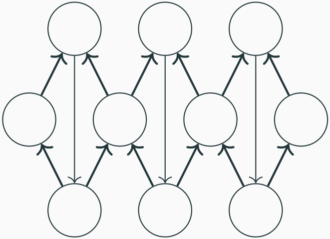
Reduction from Multicolored Clique: given G with $V(G) = V_1 \uplus \cdots \uplus V_k$, find a clique with one vertex in each V_i.



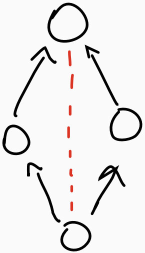
W[1]-hardness of Directed Symmetric Multicut

Diamond

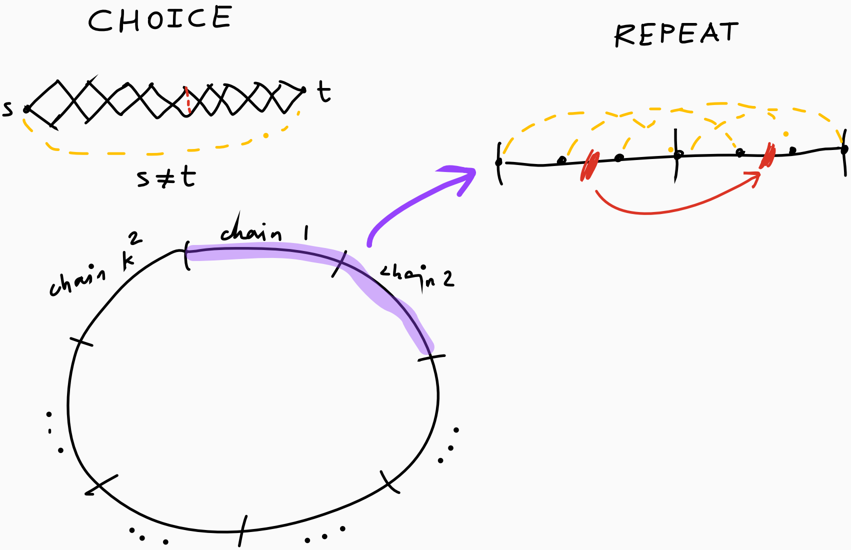




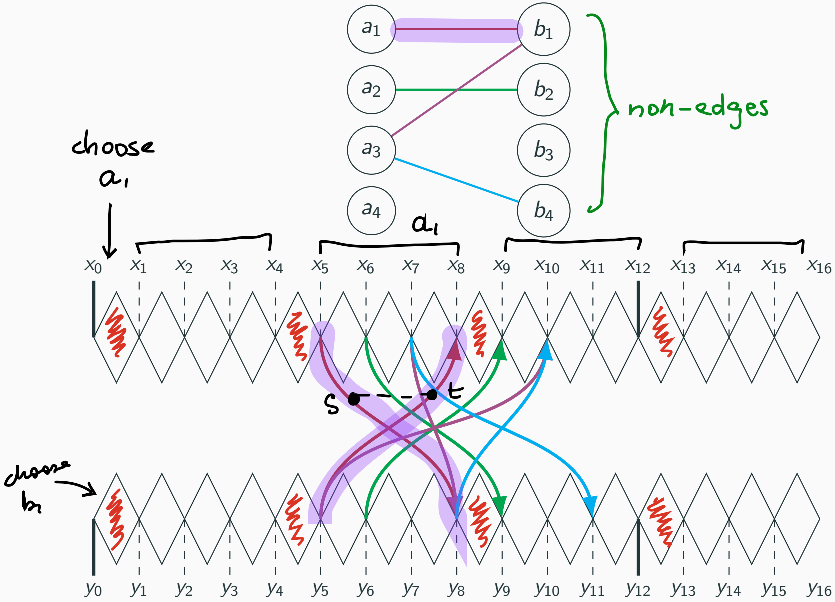
chain



W[1]-hardness of Directed Symmetric Multicut



W[1]-hardness of Directed Symmetric Multicut



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Ex.: $(x = y) \equiv \neg(x < y) \wedge \neg(x > y)$.
Ex.: $\text{Between}(x, y, z) \equiv (x < y \wedge y < z) \vee (z < y \wedge y < x)$.

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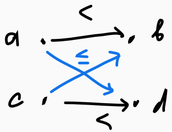

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Obstacle: $\textsc{MinCSP}((a < b) \wedge (c < d) \wedge (a \leq d) \wedge (c \leq b))$.

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Thank you!